EETBD 415
Bode Phase Plot Example

$$G(s) = \frac{500(s+50)}{(s+10)(s+100)(s+500)}$$

$$\angle G(j\omega) = \angle(\text{numerator}) - \angle(\text{denominator})$$

1. Find all corner frequencies: ω_c = 10, 50, 100, 500 rad/s
2. Find all decade above/decade below frequencies (in order):

$0.1\omega_c$ (rad/s)	$10\omega_c$ (rad/s)	$\angle G(j\omega)$	Notes
1		0	Phase angle starts at 0° (no "s" term)
5		-26.56°	Slope is at -45°/decade till 5 rad/sec
10		-26.56°	No slope between 5 and 10 rad/sec
50		-60.25°	Slope is at -45°/decade till 50 rad/sec
	100	-82.88°	Slope is at -90°/decade till 100 rad/sec
	500	-123.69°	Slope is at -45°/decade till 500 rad/sec
	1000	-153.43°	Slope is at -90°/decade till 1000 rad/sec
	5000+	-180°	Slope is at -45°/decade till 5000 rad/sec.

3. Find the phase angle for each component in the transfer function, noting that:
   - The phase angle is assumed zero for a component when $\omega < 0.1\omega_c$
   - The phase angle is assumed 90° for a component when $\omega > 10\omega_c$
   - The phase angle for an "s" term (corner frequency of zero) is 90°

(a) $0 < \omega < 1$ rad/s: $\angle G(j\omega) = (0 + 0) - (0 + 0 + 0) = 0^\circ$

(b) $\omega = 5$ rad/s $\angle G(j\omega) = (0+0) - (26.56 + 0 + 0) = -26.56^\circ$

(c) $\omega = 50$ rad/s $\angle G(j\omega) = (0+45) - (78.69 + 26.56 + 0) = -60.25^\circ$

(d) $\omega = 100$ rad/s $\angle G(j\omega) = (0+63.43) - (90 + 45 + 11.31) = -82.88^\circ$

(e) $\omega = 500$ rad/s $\angle G(j\omega) = (0+90) - (90 + 78.69 + 45) = -123.69^\circ$

(f) $\omega = 1000$ rad/s $\angle G(j\omega) = (0+90) - (90 + 90 + 63.43) = -153.43^\circ$

Above, 5000 rad/sec, the slope is zero. The final phase angle (at frequencies above 10x the highest corner frequency) can be found from the following equation:

$$\angle G(j\omega, \omega \text{ large}) = -90 \text{ (\#poles - \#zeros) degrees}$$

The Bode phase plot for this transfer function is seen as follows: